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Inflation During and After the Zero Lower Bound *

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I Introduction

The zero lower bound (ZLB) for nominal interest rates constrains monetary policy responses to adverse shocks. This inability to stabilize the economy is a major concern of central bankers. Because Japan experienced a long period of zero interest rates accompanied by falling prices from the late 1990s to the present, central bankers are also concerned about the possibility of deflation. This paper studies inflation dynamics at the ZLB and during an exit from the ZLB.

First, we compare and contrast the experiences of the three largest economies in which interest rates reached the ZLB in recent years: Japan, the United States, and the Euro Area in Section II. This comparison reveals important qualitative differences. In Japan, which has

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been at the ZLB since 1999 except for two brief stints, inflation has been negative, long-run inflation expectations demonstrate significant fluctuations, the and the real interest rate has remained positive. During the 2009-2015 ZLB episode in the U.S., on the other hand, except for two quarters early on, inflation has been positive but real rates have been consistently negative. Inflation and real rates in the Euro Area have behaved qualitatively similar to the U.S. Another crucial difference between the U.S. and Europe on the one hand, and Japan on the other is the remarkable stability of long-run inflation expectations in the former two economies despite fairly large swings in actual inflation. Using a flexible time-series model with a good inflation forecasting record, we extract a low frequency trend-inflation component, which remains positive in the U.S. and the Euro Area throughout the sample, but has been negative in Japan since the late 1990s. Looking into the future, the time series model predicts a substantial probability of deflation for Japan over the next five years, while for the U.S. and Europe these probabilities are no more than 20%.

Second, we turn to one of the workhorse models for monetary policy analysis to understand the differences among the three economies and to study a possible exit from the ZLB: a textbook-style New Keynesian dynamic stochastic general equilibrium (DSGE) model with ZLB constraint. It is well known that the ZLB generates multiple equilibria: the model predicts a set of different economic outcomes conditional on the same set of fundamentals.[1] Of course, in reality only one of these outcomes is observed. Thus, it is common to augment a model that lacks a unique equilibrium with a probabilistic selection mechanism, which is often called a sunspot shock. This sunspot shock is a placeholder for a more complete theory of how firms and households coordinate their beliefs and actions.

Multiplicity of equilibria is both a blessing and a curse. As we demonstrate, it is a blessing for empirical researchers who are trying to explain very different macroeconomic experiences, say in the U.S. and Japan, with a single economic model. Unfortunately, it may turn out to be a curse for policy makers, because the same monetary policy action of, say, changing interest rates or making announcements about targeted inflation rates, may have very different effects, depending on the equilibrium. However, there is also an opportunity for policy making: actions and statements of central banks may influence the coordination of beliefs among private sector agents and lead to the selection of a desirable equilibrium. Moreover, one can attempt to design policies that make some of the equilibria, preferably the undesirable ones, unsustainable. While the model considered in this paper is not rich enough to provide a formal analysis of equilibrium selection through central bank actions, we will offer an informal assessment.

Section III starts by reviewing the main building blocks of New Keynesian DSGE models: the consumption Euler equation, the New Keynesian Phillips curve (NKPC), and the monetary policy rule. The Phillips curve has recently been criticized because, using a backward-looking Phillips curve, one would have predicted a strong deflation in the U.S. for the period of 2009 to 2012 based on empirical measures of output and unemployment gaps. This, of course, is not what happened in the U.S.. We review some recent research that shows that the criticism is unjustified: once one correctly accounts for the forward-looking nature of the NKPC, the New-Keynesian model provides a good description of the U.S. ZLB experience. We proceed by reviewing various types of equilibria that can arise in New Keynesian DSGE models with ZLB constraints, starting with the analysis of steady states and perfect-foresight dynamics. Based on our work in Aruoba, Cuba-Borda, and Schorfheide (2014), henceforth ACS, we construct a stochastic equilibrium in which the economy may alternate between a targeted-inflation and a deflation regime. This specification is used for the subsequent quantitative analysis.

In Section IV we confront our quantitative model with data from the three economies. Looking at inflation rates, inflation expectations, and interest rates, the Japanese ZLB experience seems more consistent with the deflation regime while U.S. data appear to be consistent with the targeted-inflation. While too early to tell, so far the European experience is also consistent with the targeted-inflation regime. How costly is it to be trapped in what we call a deflation regime? We discuss potential macroeconomic costs of low inflation rates in Section V. Multiplicity of equilibria generates ambiguity for policy makers, who, ideally, want to know precisely how its actions are linked to outcomes so that he can choose the best action out of a number of feasible ones. One natural response is to consider policies that eliminate the multiplicity and thereby make it easier to predict the effects of macroeconomic policies. We discuss some of these policies in Section VI.

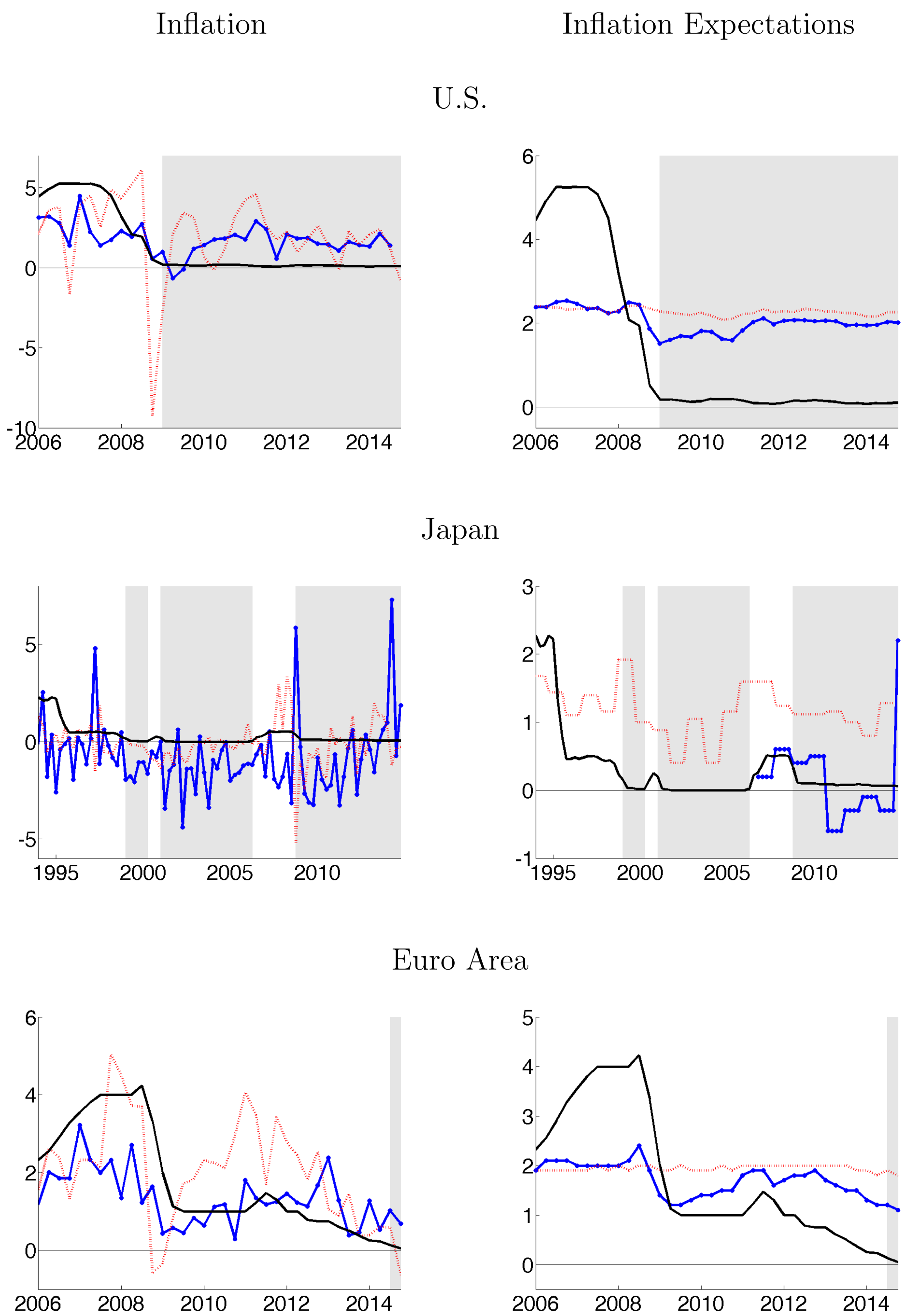
More concretely, we provide a quantitative assessment of an increase in the target inflation, which has been proposed by several prominent policy makers and scholars. First, we discuss the implications of a historical counterfactual where the Federal Reserve adopted a 4% inflation target in 1984. In this scenario there could be some improvements in welfare, especially if the Federal Reserve acts even more aggressively to cut the policy rates. Our results show that recovery from the Great Recession would have been about a year shorter. Second, we have the Federal Reserve change their target abruptly in 2014, in the middle of the ZLB episode in the U.S., which is of course the more realistic experiment. Our findings show that this policy change does not generate clear short- to medium-run benefits. The long-run benefits (or costs) strongly depend on the likelihood of adverse shocks that push the economy to the ZLB yet again. Section VII provides a brief conclusion. Data definitions, parameter estimates, and other technical details are relegated to the Appendix.

II Inflation in the U.S., Japan, and the Euro Area

The empirical analysis in this paper will focus on the recent experiences of the U.S., Japan, and the Euro Area. Figure 1 depicts inflation rates and inflation expectations for these three economies. Precise data definitions are provided in Appendix A. The panels on the left depict the monetary policy interest rate as well as two inflation rates: gross domestic production (GDP) deflator inflation and consumer price index (CPI) inflation. Most of the subsequent analysis will be based on GDP deflator inflation, which is the inflation rate that is typically used in the estimation of DSGE models. We include CPI inflation, which tends to be a bit more volatile, at least in the U.S. and the Euro Area, because the inflation expectations depicted in the panels on the right refer to changes in the consumer prices.

Interest rates in the U.S. reached the ZLB in 2009. The policy rate of the Bank of Japan has been essentially zero since 1999, with the exception of a short period in 2000-2001 and 2007-08 when the policy rate increased to roughly 50 basis points (bp). Interest rates in the Euro Area have been below 50 bp since 2012:Q2 and effectively reached zero in 2014:Q3. Several observations from Figure 1 stand out. First, while in the U.S. the ZLB episode is associated with positive inflation, GDP deflator inflation rates in Japan have been

Figure 1: Inflation and Inflation Expectations



Notes: Left panels: monetary policy interest rate (solid black), CPI inflation (dotted red), GDP deflator inflation (solid-dotted blue), where the latter two are annualized quarterly rates. Right panels: monetary policy interest rate (solid black), 5-year-ahead (10-year-ahead for Japan) inflation expectations (dotted red), 1-year-ahead inflation expectations (solid-dotted blue). The shaded gray intervals characterize the ZLB episodes.

negative, with the exception of two short spikes.[2] Second, the verdict on the Euro Area is still out: inflation rates have been falling toward the end of the sample as the policy rate has approached zero.

Third, long-run (5-year-ahead) inflation expectations have been remarkably stable in the U.S. and the Euro Area, despite falling policy rates. Even more remarkable, 10-year-ahead inflation expectations in Japan have stayed around 1% even the average inflation rate over the past 15 years was negative. Short-run inflation expectations appear to be more sensitive to economic conditions. In the U.S. they started to fall in 2008:Q4 as the economy was experiencing a major disruption in the financial sector. However, at quarterly frequency they never dropped below 1.5% and climbed to 2% by 2011:Q1, which is consistent with the evolution of actual inflation. In the Euro Area, prolonged drops in the policy rate are associated with a fall in the 1-year-ahead inflation expectations but at the end of 2014, short-run inflation expectations are still above 1%.

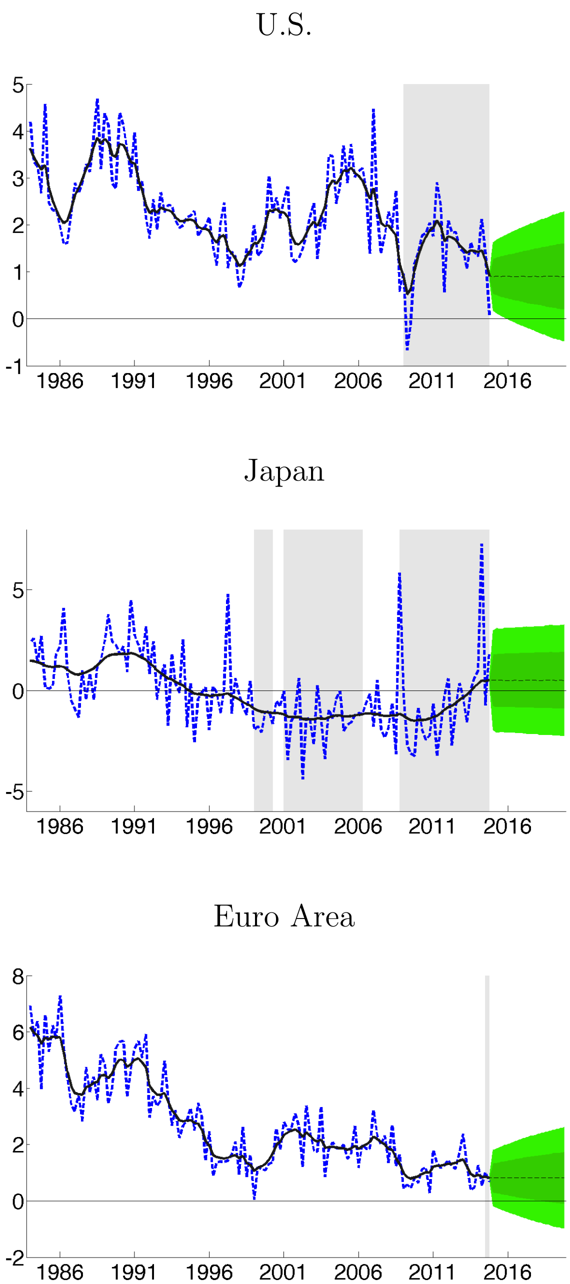
Underlying the inflation expectations data are a variety of econometric forecasting models which in many cases are adjusted by the judgment of the individual(s) publishing the forecast. In the remainder of this section, we fit a small time series model to the GDP deflator inflation series plotted in Figure 1. This model serves two purposes: we use it to extract a low-frequency trend component from the inflation series and we generate probability density forecasts conditional on data until 2014:Q4. Our econometric model of choice is the following univariate unobserved components model proposed by Stock and Watson (2007):

$$\begin{aligned} \pi_t &= \tau_t + \sigma \exp(h_{\epsilon,t})\epsilon_t, \\ \tau_t &= \tau_{t-1} + (\varphi\sigma)\exp(h_{\eta,t})\eta_t \\ h_{j,t} &= \rho_j h_{j,t-1} + \sqrt{1-\rho_j^2}\sigma_{v_j} v_{j,t}, \quad j \in \{\epsilon, \eta\}. \end{aligned} \tag{1}$$

The model decomposes the inflation series in a local-level component, τ_t, and serially uncorrelated short-run fluctuations, ϵ_t. The innovations associated with the local-level process and the short-run fluctuations exhibit stochastic volatility to account for the fact that the degree of time variation in the low frequency component and the importance of the short-run fluctuations for the inflation dynamics may change over time. Notice that the h-step-ahead point forecast from this model is simply the filtered estimate of the local-level component $\hat{\tau}_{t|t} = \mathbb{E}[\tau_t|\pi_{1:t}]$ for all h, where $\pi_{1:t}$ denotes the sequence $\{\pi_1, \dots, \pi_t\}$. While the model cannot capture the divergence of short-run and long-run inflation expectations evident in Figure 1, Stock and Watson (2007) and, more recently, Faust and Wright (2013) show that it is a competitive forecast model that extrapolates past inflation rates into the future in a way that is more accurate than many of its competitors over most horizons. The local-level model captures two features that are important for inflation forecasting: time-variation in trend inflation through τ_t and time variation in the persistence of inflation through the relative magnitude of the log volatilities $h_{\epsilon,t}$ and $h_{\eta,t}$. Its estimation is described in Appendix B.

Figure 2 depicts the filtered local-level process $\hat{\tau}_{t|t}$ as well as density forecasts for the period 2015:Q1 to 2019:Q4. $\hat{\tau}_{t|t}$ tracks the low frequency moments of inflation. For reasons that will become apparent in Section III we refrain from interpreting τ_t as the central bank's target inflation rate. We simply call it trend inflation. For the U.S. and the Euro Area trend

Figure 2: Local Level Processes for Inflation



Notes: Each panel depicts GDP deflator inflation (dashed blue) and filtered estimates (solid black) of the low frequency component of inflation as measured by the local-level component τ_t in (1). The local-level models are estimated based on data from 1984:Q1-2014:Q4. The shaded green bands characterizes 20-step-ahead predictive distribution, using 2014:Q4 as forecast origin (median, 60%, and 90% predictive intervals). The shaded gray intervals characterize the ZLB episodes.

inflation clearly has been positive until 2014:Q4, whereas it has been negative in Japan since 1996.

The shaded areas starting in 2015:Q1 indicate 60% and 90% predictive intervals obtained from the local-level model. Note that trend inflation evolves according to a random walk. This means that the point prediction stays constant over time, but the prediction intervals widen. Over time, uncertainty about trend inflation dominates uncertainty about the short-run fluctuations. For the U.S. and Euro Area the short-run fluctuations have been fairly stable recently and the uncertainty about trend inflation is apparent in the widening interval predictions. For Japan, uncertainty about short-run fluctuations caused by a recent spike in inflation volatility is the main contributor to uncertainty about future inflation. According to the forecasts from the local-level model the risk of experiencing deflation over the next five years remains close to 50% for Japan. For the U.S. it increases from essentially zero in the short-run to about 15% in five years from now. Finally, the Euro Area is in-between Japan and the U.S. In the short-run the risk of deflation is about 5% and it increases to about 20% over the next five years. While the model does not use any economic theory or information from other macroeconomic indicators, it is important to keep in mind that models like this generate on average very reliable forecasts.

III Inflation in New Keynesian DSGE Models

In the remainder of this paper we look at inflation dynamics through the lens of a small-scale New Keynesian DSGE model. Since the influential work of Smets and Wouters (2003) central banks around the world started to include estimated DSGE models into the suites of econometric models that are used to generate projections and support policy decisions. Although these models abstract from the complexities of modern-day economies, they provide a useful framework to understand the dynamics of output, inflation, and interest rates as well as the potential effects of monetary and fiscal policy interventions. While the Great Recession of 2007-09 has triggered a lot of research on how to incorporate financial and labor market frictions into DSGE models and how to model unconventional monetary policy, we work with a fairly rudimentary version of a New Keynesian DSGE model and focus on some fundamental mechanisms that are also part of richer DSGE models. We first review the key model elements (Section III.A) and then discuss various types of equilibria that can arise in these models (Section III.B). Each equilibrium is associated with distinct implications for inflation dynamics.

III.A Key Model Elements

New Keynesian DSGE model comprises three main elements: a consumption Euler equation that links interest rates to consumption and economic activity more generally; a New Keynesian Phillips curve (NKPC) that links inflation to expectations about current and future marginal costs, and hence real activity; and monetary and fiscal policy rules that determine interest rate and taxes conditional on the state of the economy. In turn, we will review each of these elements and examine the data from the perspective of these equilibrium relationships. A fully specified small-scale DSGE model that encompasses these elements is presented in Appendix C. We assume that time is discrete and that length of a period t is three months.

III.A.i Consumption Euler Equation

Households are assumed to derive utility from consumption and leisure. The maximization of the expected sum of discounted future utility with respect to the choice of consumption leads to the following inter-temporal first-order condition:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\delta_{t+1}}{\delta_t} \right) Q_{t+1|t} \frac{R_t}{\pi_{t+1}} \right]. \tag{2}$$

Here β is the average discount factor, $Q_{t+1|t}$ is the ratio of the marginal utilities of consumption in periods $t+1$ and t, R_t is the gross nominal interest rate, and π_t is the gross inflation rate. Finally, δ_t captures exogenous fluctuations in the discount factor for period t utility. The process δ_t plays an important role in capturing movements in the real interest rate. It is convenient to define the stochastic discount factor

$$M_{t+1} = \beta \left(\frac{\delta_{t+1}}{\delta_t} \right) Q_{t+1|t}, \tag{3}$$

which can be used to price any asset in the economy. Consider, for instance, a risk-free asset that generates a real return r_t^f between period t and $t+1$. The return r_t^f has to satisfy

$$1 = \mathbb{E}_t[M_{t+1} r_t^f], \tag{4}$$

which leads to

$$r_t^f = \mathbb{E}_t \left[\frac{1}{M_{t+1}} \right]. \tag{5}$$

In the model economy described in Appendix C, the stochastic discount factor takes the specific form

$$M_{t+1} = \beta \left(\frac{\delta_{t+1}}{\delta_t} \right) \left(\frac{C_{t+1}/C_t}{\gamma z_{t+1}} \right)^{-\tau}, \tag{6}$$

where C_t is consumption, γz_{t+1} is the (stochastic) technology growth rate in the economy and $\tau > 0$ reflects the degree of risk aversion. The stochastic discount factor is high in period $t+1$ if $\delta_{t+1} > \delta_t$ or consumption growth is low relative to productivity growth. The pricing equation (4) implies that a high stochastic discount factor is associated with low real returns.

Throughout this paper we often refer to steady states and log-linear approximations around steady states. In our notion of steady state, appropriately detrended model variables are constant over time (which we denote by replacing the t subscript with a $*$ subscript) and the economy is not perturbed by any exogenous stochastic shocks. A log-linearization around a steady state refers to an approximation of $f(x_t)$ through a first-order Taylor expansion in terms of $\ln x_t$ around $\ln x_*$. We use the notation $\widehat{x}_t = \ln(x_t/x_*)$. Combining (2) and (5), we obtain the following steady state relationship between the ex-ante real rate, the nominal interest rate, and the inflation rate (Fisher equation)

$$r^f_* = \frac{R_*}{\pi_*}. \tag{7}$$

A log-linearization approximation yields

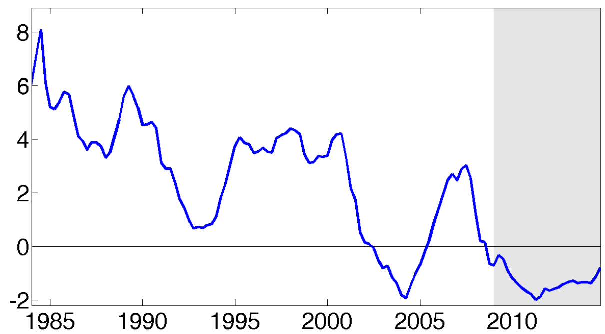
$$\widehat{r}^f_t = \widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}]. \tag{8}$$

Both (7) and (8) play a central role in the subsequent analysis.

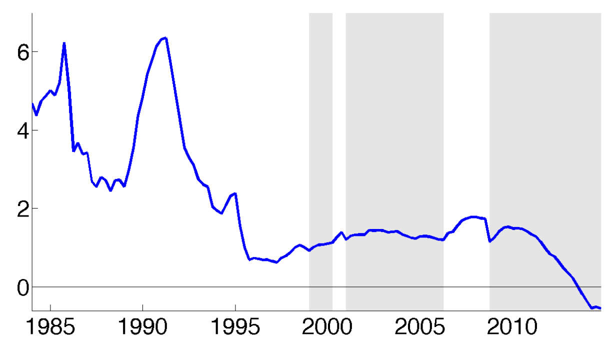
Figure 3 plots implied ex-ante real interest rates (in annualized percentages) based on (7) and (8). The one-step-ahead inflation forecasts $\mathbb{E}_t[\widehat{\pi}_{t+1}]$ are obtained from the local-level model (1) as the filtered estimates $\mathbb{E}[\tau_t|\pi_{1:t}]$. The most striking difference between the U.S. and the Euro Area on the one hand and Japan on the other hand is that the implied real

Figure 3: Ex Ante Real Interest Rates

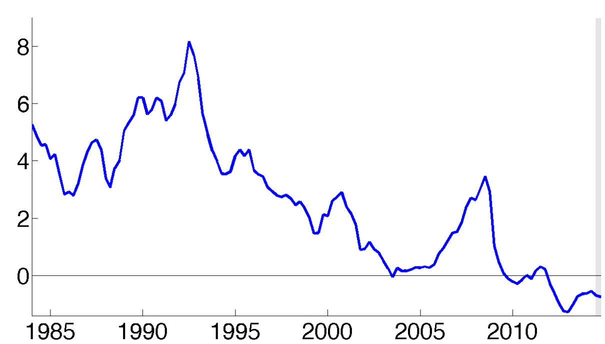
U.S.



Japan



Euro Area



Notes: Each panel depicts ex-ante real interest rates computed as $400 \ln r_t^f = 400(\ln R_t - \mathbb{E}_t[\ln \pi_{t+1}])$. The inflation expectations are computed from the local-level model (1) and defined as the filtered estimates of τ_t. The shaded gray intervals characterize the ZLB episodes.

interest rate in Japan has stayed positive throughout the ZLB episode until 2013:Q3, whereas it has been negative in the U.S. since 2008:Q4 and the Euro Area since 2009:Q4 (with the exception of 2011). According to (5) and (6), a negative real rate is associated with an expectation of consumption growth that is below the trend growth rate. However, observed consumption growth in the U.S. is not sufficiently low to generate a persistent negative real rate, which means that in fitting the data, the discount factor shock δ_t will play an important role.

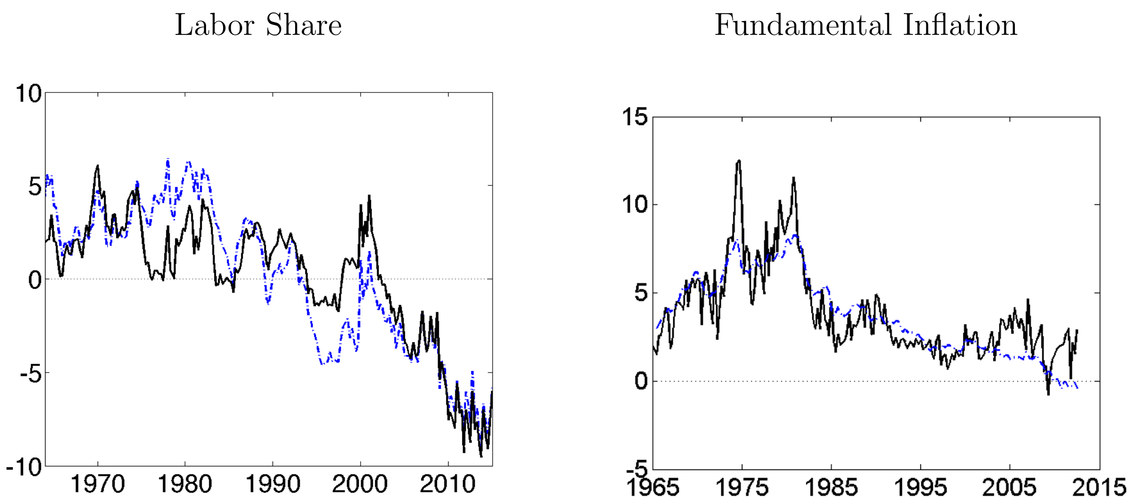
III.A.ii New Keynesian Phillips Curve

The NKPC provides a link between inflation and real activity. It is typically derived under the assumption that production takes place in two stages. In the first stage, monopolistically competitive intermediate goods producers utilize labor and other factors of production, e.g., capital, to produce their goods. Each producer is facing a downward sloping demand curve and costs of adjusting nominal prices, which generates price stickiness. The intermediate goods are purchased by perfectly competitive final-goods-producing firms which simply turn the intermediate goods into an aggregate good that can be used for consumption, investment, or government spending.

The resulting equilibrium condition that describes the profit-maximizing prices set by the intermediate goods producers is called NKPC. A log-linear approximation around a level of inflation, assuming price adjustments at that rate are costless, takes the form:

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{mc}_t + \lambda_t, \tag{9}$$

Figure 4: Marginal Costs and Fundamental Inflation



Notes: The left panel depicts two labor share series in percentage deviations from their mean: solid black line is nonfarm business sector labor share (Source: FRED); dashed blue line is the product of compensation per hour (nonfarm business sector), civilian employment (sixteen years and over), and average weekly hours (private industries) divided by GDP (Source: Haver Analytics). The right panel depicts GDP deflator inflation (solid black line) and fundamental inflation (dashed blue line) from a medium-scale DSGE model with financial frictions (Source: Del Negro, Giannoni, and Schorfheide (2015)).

where κ is the slope of the Phillips curve, $\widehat{mc}_t$ is marginal costs and λ_t is an exogenous price mark-up shock that sometimes is added to improve the empirical fit of the NKPC. The key feature of this version of the Phillips curve is that it is forward looking: current inflation depends on current real activity (through marginal costs) and expected inflation in the next period.

Many of the standard DSGE models, e.g., the widely-referenced Smets and Wouters (2007) model as well as the small-scale DSGE model described in Appendix C, imply that marginal costs are proportional to the labor share, which can be measured in the data. The left panel of Figure 4 depicts two measures of the labor share in the U.S in percentage deviations from a mean computed over the period 1964:Q1 to 2015:Q1 . The labor share has been fairly stable until 2002 and has exhibited a downward trend since then that continued during and after the Great Recession. It is apparent from (9) that, ceteris paribus, a drop in marginal costs generates deflationary pressure. How much depends on the details of the model. If the downward trend is generated by a shift of the steady state it may not affect inflation at all, because the NKPC in (9) characterizes fluctuations around a steady state or long-run trend. Most importantly, expectations about future marginal costs are very important, which we will discuss in more detail below.

The NKPC has been recently criticized by prominent macroeconomists, e.g., Ball and Mazumder (2011) and Hall (2011), because the absence of deflation in the U.S. in the aftermath of the Great Recession (see Figure 2) seems to be inconsistent with the drop in marginal costs in the left panel of Figure 4. For instance, Ball and Mazumder (2011) estimate a backward-looking Phillips curve (the term $\mathbb{E}_t[\widehat{\pi}_{t+1}]$ in (9) is replaced by lags of $\widehat{\pi}_t$) based on data from 1960 to 2007 and then predict inflation conditional on observed measures of economic slack for 2008-2010. Given the drop in marginal costs (and a measure of the output gap) the backward-looking Phillips curve predicts deflation as high as 4%, which did not happen. Thus, from the perspective of a backward-looking Phillips curve, there is a missing disinflation puzzle in the U.S.

However, the NKPC that underlies the current generation of DSGE models is forward-looking. Solving (9) forward under the assumption that the mark-up shock process is AR(1) with autoregressive parameter ρ_λ we obtain

$$\widehat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[\widehat{mc}_{t+j}] + \frac{1}{1+\rho_\lambda \beta}\lambda_t. \tag{10}$$

The first sum is called fundamental inflation. The right panel of Figure 4 shows the fundamental inflation series constructed by Del Negro, Giannoni, and Schorfheide (2015). It is based on an estimated version of the Smets and Wouters (2007) model with financial frictions and tracks the low frequency component of inflation well. Del Negro, Giannoni, and Schorfheide (2015) also document that their DSGE model is able to predict the observed path of inflation quite accurately from 2008:Q4 onward. Part of the reason is that despite the fall of the labor share toward the end of the sample, fundamental inflation does not become negative during and after the Great Recession because agents in the model expect marginal costs to rise again in the near future. Coibion and Gorodnichenko (2015) estimate forward-looking Phillips curves along the line of (9) by using survey expectations as proxies for expected inflation. They find that a deflation in 2009 - 2011 is avoided by high inflation expectations relative to current inflation due to, among other factors, an increase in energy prices and a preceding decline in inflation in early 2009.

III.A.iii Monetary Policy and Fiscal Policy

Monetary policy in DSGE models is typically described through an interest feedback rule. Because the ZLB constraint is an important part of our analysis we introduce it explicitly as follows:

$$R_t = \max\left\{1,\ \bar{R}_t e^{\epsilon_{R,t}}\right\}. \tag{11}$$

Here $\epsilon_{R,t}$ is an unanticipated monetary policy shock that captures deviations from the systematic part of the interest rate feedback rule, $\bar{R}_t$. $\bar{R}_t$ is determined as a function of the current state of the economy. We assume that

$$\bar{R}_t = \left(r_*^f \bar{\pi} \left(\frac{\pi_t}{\bar{\pi}}\right)^{\psi_1} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\psi_2} \right)^{1-\rho_R} R_{t-1}^{\rho_R}, \tag{12}$$

where $\bar{\pi}$ is the targeted inflation rate and $\bar{Y}_t$ is the target level of output. In theoretical studies the targeted level of output often corresponds to the level of output in the absence of nominal rigidities and mark-up shocks because from an optimal policy perspective, this is the level of output around which the central bank should stabilize fluctuations. However, it appears that in reality the behavior of central banks is well described by trying to keep output close to official measures of potential output, which can be approximated by a slow-moving trend. Thus, throughout this paper we use exponential smoothing to construct $\bar{Y}_t$ directly from historical output data. It is given by

$$\ln \bar{Y}_t = \alpha \ln \bar{Y}_{t-1} + (1 - \alpha) \ln Y_t + \alpha \ln \gamma. \tag{13}$$

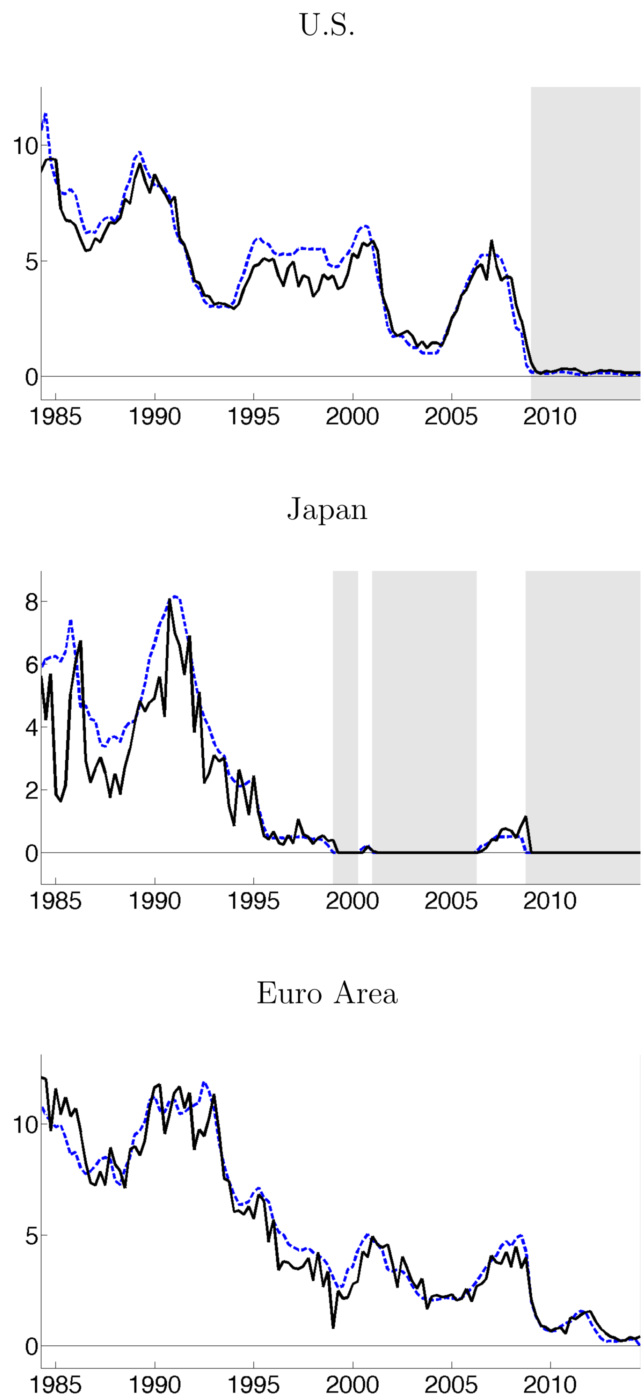
The definition of $\bar{R}_t$ is such that conditional on the monetary policy rule coefficients, it can be directly computed from the data. We plot $\bar{R}_t$ in Figure 5. We calibrate α to match official measures of potential output and fix $\psi_1 = 1.5$ and $\psi_2 = 0.1$. These values are close to the classic Taylor rule coefficients. The interest rate smoothing coefficient is estimated along with other DSGE model coefficients in preparation for the analysis in the remaining sections of this paper. In general $\bar{R}_t$ tracks the actual interest rate fairly well, even during the ZLB episodes.

In addition to the monetary policy rule, we also need to specify a fiscal policy. We write the government budget constraint in real terms as

$$G_t + R_{t-1} \frac{1}{\pi_t} \frac{B_{t-1}}{P_{t-1}} = \frac{T_t}{P_t} + \frac{B_t}{P_t}, \tag{14}$$

where G_t is an exogenous spending process, B_t is nominal government debt, and T_t are nominal taxes or transfers. Government spending, debt, and taxes, may react to the state of

Figure 5: Monetary Policy Rates



Notes: Each panel depicts the monetary policy interest rate (solid black line, see Appendix A for data definition) and the systematic part of the desired interest rate $\bar{R}_t$ (dashed blue line), see (12) for definition. The shaded gray intervals characterize the ZLB episodes.

the economy. In most monetary DSGE models it is assumed that government spending as a fraction of GDP is exogenous and that the government uses lump-sum taxes and transfers to balance the budget. Because the exact nature of the response of the fiscal authority to the state of the economy has important consequences for the multiplicity of equilibria, we will postpone a more detailed discussion.

III.A.iv Small-Scale versus Large-Scale Models

In the preceding sections we sketched the key building blocks of New Keynesian DSGE models. Appendix C contains the remaining missing pieces to turn these building blocks into a coherent small-scale DSGE model. The literature has developed much richer medium- and large-scale DSGE models. To give a few examples, the models estimated by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) contain capital as a factor of production and feature habit formation in consumption, investment-adjustment costs, variable capital utilization and wage rigidity. The models of Christiano, Motto, and Rostagno (2003) and Gertler and Kiyotaki (2010) prominently feature financial frictions. The models of Gertler, Sala, and Trigari (2008) and Christiano, Eichenbaum, and Trabandt (2013) include labor market frictions. The models of Chen, Curdia, and Ferrero (2012) and Gertler and Karadi (2011) are designed to study the effects of unconventional monetary policies. In the remainder of this paper we will proceed with a small-scale DSGE model because many of the calculations are more transparent, while it is still sufficiently rich to be used to track output, consumption, inflation, and interest rates from the U.S., Japan, and the Euro Area.

III.B ZLB and Multiplicity of Equilibria

This paper focuses on inflation dynamics when economies are at the ZLB or they exit the ZLB. In the previous section we explored some partial equilibrium implications of the NKPC, which determines inflation as a function of (future expected) marginal costs. However, the NKPC relationship is present regardless of whether the ZLB is binding or not. Thus the main reason for obtaining different inflation dynamics in periods in which the ZLB is active is that marginal cost dynamics change.[3] Nonetheless, analyzing inflation dynamics with a New Keynesian DSGE model should be straightforward. Simply solve the model subject to the ZLB constraint and simulate inflation trajectories during and after ZLB episodes. Unfortunately, the presence of multiple equilibria generates complications and implies that these DSGE models predict a wide range of inflation and real activity outcomes.

We proceed by examining the multiplicity of steady states, then we study perfect foresight dynamics, and finally we consider a stochastic equilibrium in which the economy is perturbed by exogenous shocks. The subsequent quantitative illustrations are based on a version of the DSGE model described in Appendix C, in which we consider log utility $\tau = 1$, and infinite Frisch labor supply elasticity $\eta = \infty$. We also simplify the monetary policy rule by setting $\psi_2 = \rho_R = 0$. The remaining parameters are chosen according to Table A-3.

III.B.i Steady States

The existence of two steady states can be easily seen by combining (7) with a steady state version of the simplified monetary policy rule:

$$R_* = \max \left\{ 1,\ \left(\frac{\pi_*}{\bar{\pi}} \right)^{\psi_1} \right\}. \tag{15}$$

There exist two solutions to this system of equations. The first solution is called the targeted-inflation steady state:

$$R_* = r_*^f \bar{\pi}, \quad \pi_* = \bar{\pi}. \tag{16}$$

Here the steady state inflation rate equals the inflation rate targeted by the central bank. The second solution, in which the net nominal interest rate is zero and inflation is negative, is called the deflation steady state:

$$R_* = 1, \quad \pi_* = \frac{1}{r_*^f}. \tag{17}$$

In both steady states the real interest rate is given by $r_*^f = \gamma/\beta$. Moreover, both steady states are fiscally sustainable under a passive fiscal policy that balances the budget using lump-sum taxes. The real value of government debt can be kept stable at $(B/P)_*$ and the real interest rate payments on the debt are constant at $(r_*^f - 1)(B/P)_*$. Notice, however, that the nominal value of government debt will change over time, depending on the steady state inflation rate π_*.

An important question is whether households are better off in one steady state or another. The answer depends on various auxiliary assumptions and will be explored in more detail in Section V. A casual look at the data in Figures 1 and 3 suggest that Japan's experience of zero nominal interest rates, deflation, and positive real rates is consistent with the deflation steady state. The U.S. experience of negative real rates does not seem to be consistent with either steady state.

III.B.ii Perfect Foresight Dynamics

The analysis of steady states does not provide any insights into inflation dynamics. We proceed by exploring some of the dynamic properties of our DSGE model. For now, we abstract from uncertainty about the realization of exogenous shock processes and assume that agents have perfect foresight. We take a log-linear approximation of the three key model equations around the targeted-inflation steady state and then impose the ZLB constraint on the log-linearized monetary policy. The consumption Euler equation and NKPC curve can be written as

$$\begin{aligned} \widehat{c}_t &= \widehat{c}_{t+1} - (\widehat{R}_t - \widehat{r}_t - \pi_{t+1}) \qquad (18) \\ \widehat{\pi}_t &= \beta\widehat{\pi}_{t+1} + \kappa\widehat{c}_t, \end{aligned}$$

where $\widehat{r}_t$ can be interpreted as a real rate shock.[4] Note that under perfect foresight we can drop the expectations $\mathbb{E}_t[\cdot]$. The log-linearization of the monetary policy rule yields

$$\widehat{R}_t = \max\ \big\{ -\ln(r_*^f\bar{\pi}),\ \psi_1\widehat{\pi}_t \big\}. \qquad (19)$$

The dynamics of consumption, inflation, and interest rates have to satisfy the set of difference equations in (18) and (19). Notice that the multiplicity of steady states is still present in (18) and (19). Suppose that $\widehat{r}_t = 0$, then one time invariant solution is $\widehat{c}_t = \widehat{R}_t = \widehat{\pi}_t = 0$. The second time invariant solution is

$$\widehat{R}_t = \widehat{\pi}_t = -\ln(r_*^f\bar{\pi}), \quad \widehat{c}_t = -\frac{1-\beta}{\kappa}\ln(r_*^f\bar{\pi}), \quad \text{for all } t.$$

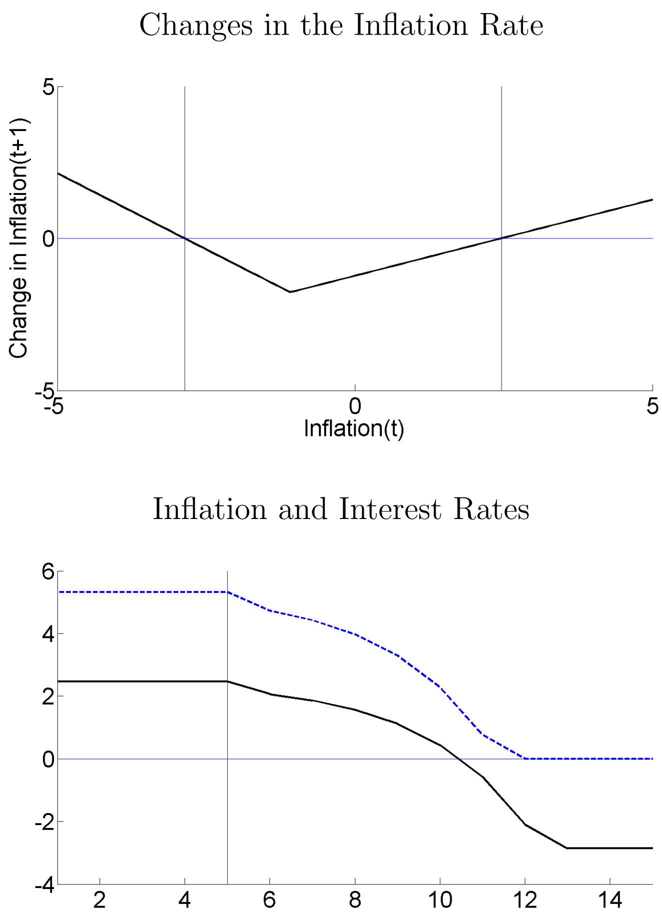
We can call the second solution the deflation steady state of the linearized system. The literature typically focuses on solutions to these difference equations that are non-explosive, because explosive dynamics tend to violate transversality conditions associated with the underlying dynamic programming problem.[5] There is a long literature that examines conditions under which the stable dynamics are unique. In the simple New Keynesian DSGE model considered here, uniqueness can be ensured by setting $\psi > 1$. This is often called active monetary policy: the central bank raises (lowers) real rates in response to inflation being above (below) its target level $\bar{\pi}$.[6] Once the ZLB binds, monetary policy becomes passive because the central bank is unable to lower interest rate in response to falling inflation rates.

Benhabib, Schmitt-Grohé, and Uribe (2001a) and Benhabib, Schmitt-Grohé, and Uribe (2001b) discuss various equilibria that can arise in the nonlinear version of a three-equation New Keynesian DSGE model. The equilibrium that has drawn a lot of attention and is of concern to policy makers is one in which the economy transitions from the targeted-inflation steady state to the deflation steady state. A casual look at the data suggests that this might describe the Japanese experience. We can illustrate these dynamics easily in the context of our linearized model. We start by assuming that prices are flexible, which implies that $\kappa = \infty$ and $\widehat{c}_t = 0$. Combining the consumption Euler equation with the monetary policy rule yields the following nonlinear difference equation for inflation

$$\widehat{\pi}_{t+1} = \max \left\{ -\ln(r_*^f \bar{\pi}),\ \psi_1 \widehat{\pi}_t \right\}. \tag{20}$$

The dynamics associated with this difference equation are depicted in Figure 6. The top panel depicts $\Delta\widehat{\pi}_{t+1}$ as a function of $\widehat{\pi}_t$. If $\Delta\widehat{\pi}_{t+1} = 0$, the system is in a steady state. The figure shows that any perturbation away from the targeted-inflation steady state will move the system away from that steady state. In particular, if inflation drops below the targeted inflation steady state, it will continue to fall and eventually settle on the deflation steady

Figure 6: Transition to the Deflation Steady State



Notes: Top panel: the vertical lines indicate the two steady states. Formally, the plot depicts $400 \ln(\pi_{t+1}/\pi_t)$ versus $400 \ln \pi_t$. Bottom panel: interest rate (dashed blue) and inflation rate (solid black) during a transition from the targeted-inflation to the deflation steady state.

state. The bottom panel shows the time path of inflation and interest rate, assuming that the system is in the targeted-inflation steady state from $t = 1$ to $t = 5$. In period $t = 6$ inflation falls and triggers the transitions to the deflation steady state.[7]

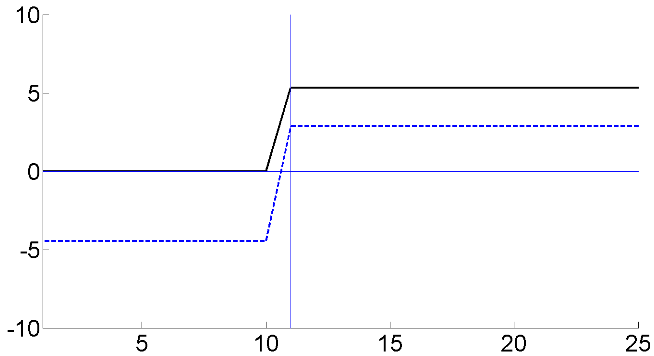
We can also use the linearized model to study a transition from the ZLB back to the targeted-inflation steady state. A common experiment conducted in the literature is to assume that an adverse real rate shock pushed the economy to the ZLB and that after a certain number of periods the economy exits from the ZLB again. To keep the analysis as simple as possible, we assume that agents know the exit date $t = T$. Figure 7 illustrates the following experiment. According to our benchmark calibration, the real interest rate and the inflation rate are 2.9% and 2.5%, respectively, in the targeted-inflation steady state. Suppose that there is an adverse real rate shock that sends the economy in the liquidy trap: $\widehat{r}_t = -7.4\%$. Simultaneously the nominal interest rate drops to the ZLB: $\widehat{R}_t = -5.4\%$. This means that $\widehat{R}_t - \widehat{r}_t = -2\%$. From period $t = T + 1$ onwards, $\widehat{r}_t$ and $\widehat{R}_t$ revert back to their steady state values. This is depicted in the top panel of the figure.

If we impose the Taylor rule (19) after $t = T$, then the only path that is non-explosive is one in which the economy reverts instantaneously to the targeted-inflation steady state, which determines $\widehat{R}_t$, $\widehat{\pi}_t$, and $\widehat{c}_t$ in periods $t > T$. For $t \leq T$ nominal interest rates are zero and output and consumption have to satisfy (18). The solution can be easily found by backward iteration: solve for time t variables as a function of time $t + 1$ variables. The resulting inflation and consumption dynamics are depicted by the red dashed lines in the center and bottom panel of Figure 7. The economy starts in a liquidity trap with deflation and low consumption caused by a negative real rate shock. Then inflation and consumption rise and eventually revert back to the targeted-inflation steady state. The longer the spell of an adverse real rate shock and zero nominal interest rates, the deeper the liquidity trap.

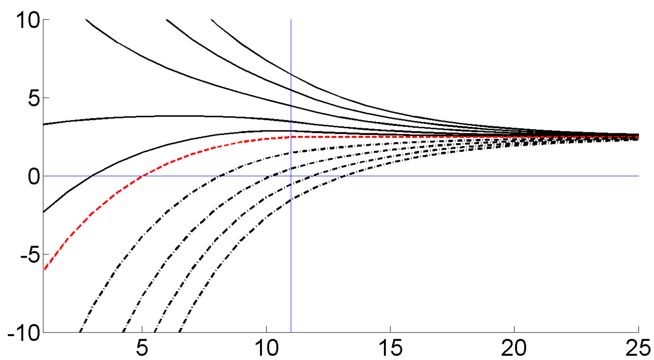
Mechanically, the potentially disastrous outcomes during the liquidity trap are due to the fact that the bivariate system (18) has one stable and one unstable root. Thus, the root that is stable during forward iterations turns unstable during backward iterations. This can generate deep contractions, but also large stimulative effects of keeping interest rate at zero

Figure 7: Perfect Foresight Dynamics in Response to a Real Rate Shock

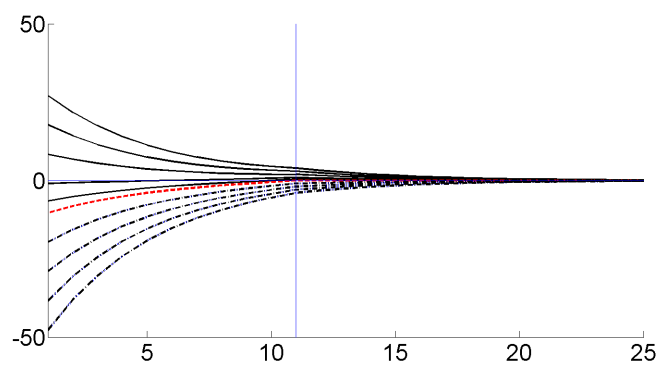
Nominal Interest Rates and Real Rate Shock (Annualized %)



Inflation Dynamics (Annualized %)



Consumption Dynamics (% Deviations from Steady State)



Notes: Top panel: solid black line is $\widehat{R}_t$; dashed blue line is $\widehat{r}_t$. Center and bottom panels: the red dashed response is obtained by imposing the Taylor rule for $t > T$. The black solid lines correspond to $\widehat{\pi}_{T+1} > 0$ whereas the black dashed-dotted lines correspond to $\widehat{\pi}_{T+1} < 0$. The vertical line indicates $t = T + 1$.

for an extended period of time as discussed, for instance, in Carlstrom, Fuerst, and Paustian (2012) and Del Negro and Schorfheide (2013).

Cochrane (2015) argues that the "standard" equilibrium generated by the interest rate rule in (19) and depicted by the dashed red lines in the center and bottom panels of Figure 7 is not the only one, and possibly not the most plausible. He constructs alternative paths for inflation and consumption, depicted with the black solid and dashed-dotted lines, by solving the bivariate system (18) forward from $T+1$ onward, imposing stability. The stability restriction determines consumption as a function of inflation in period $T+1$, which means that each equilibrium path can be indexed by $\widehat{\pi}_{T+1}$. Despite being generated conditional on the same paths of real rates and nominal interest rates, some of the alternative trajectories are associated with better inflation and consumption outcomes. This observation has a positive and a normative dimension: the red dashed path may not be the one that best describes U.S. (and possibly Euro Area) data; and good monetary policy might put the economy on a path in which inflation is positive and fairly stable and consumption does not collapse. With regard to implementation, Cochrane (2015) points out that for $t > T$, the solid black paths could be implemented using a policy rule of the form

$$\widehat{R}_t = \psi_1(\widehat{\pi}_t - \widehat{\widehat{\pi}}_t), \tag{21}$$

where $\widehat{\widehat{\pi}}_t$ is the desired inflation path. According to this policy rule the central bank conducts an equilibrium selection policy to select among the equilibria that are consistent with $\widehat{R}_t = 0$. Thus, ultimately the central bank's equilibrium-selection policy determines whether the liquidity trap will be benign or disastrous.

III.B.iii Stochastic Equilibria

While the analysis of steady states and perfect foresight equilibria can deliver important theoretical and qualitative insights, a more detailed empirical analysis requires us to examine equilibria in which the economy is perturbed by stochastic shocks. Broadly speaking, these shocks capture agents' uncertainty about future fundamentals. In the model described in Appendix C, we consider a shock to the growth rate of total factor productivity, a shock to the discount factor which generates exogenous fluctuations in the real rate, a shock to aggregate demand, and a monetary policy shock that captures unanticipated deviations from the systematic part of the interest rate feedback rule.

As we have seen previously, there are various challenges when working the New Keynesian models: multiple steady states, local indeterminacy when the ZLB is binding, and complicated nonlinearities due to an occasionally binding ZLB constraint. The subsequent empirical results are based on the computational techniques developed and applied in ACS. We use a global approximation technique to compute a stochastic equilibrium associated with the nonlinear DSGE model. To capture the multiplicity of steady states we introduce a binary exogenous sunspot shock that serves as a coordination device for agents' expectations. Depending on the realization of the sunspot shock the economy either fluctuates around the targeted-inflation steady state or around the deflation steady state. We refer to these two outcomes as targeted-inflation and deflation regime, respectively. As should be clear from our previous analysis, the two-regime equilibrium that we are constructing is by no means the only one for the nonlinear version of the DSGE model. However, it is one that we are able to characterize numerically and that generates a lot of interesting and plausible results.

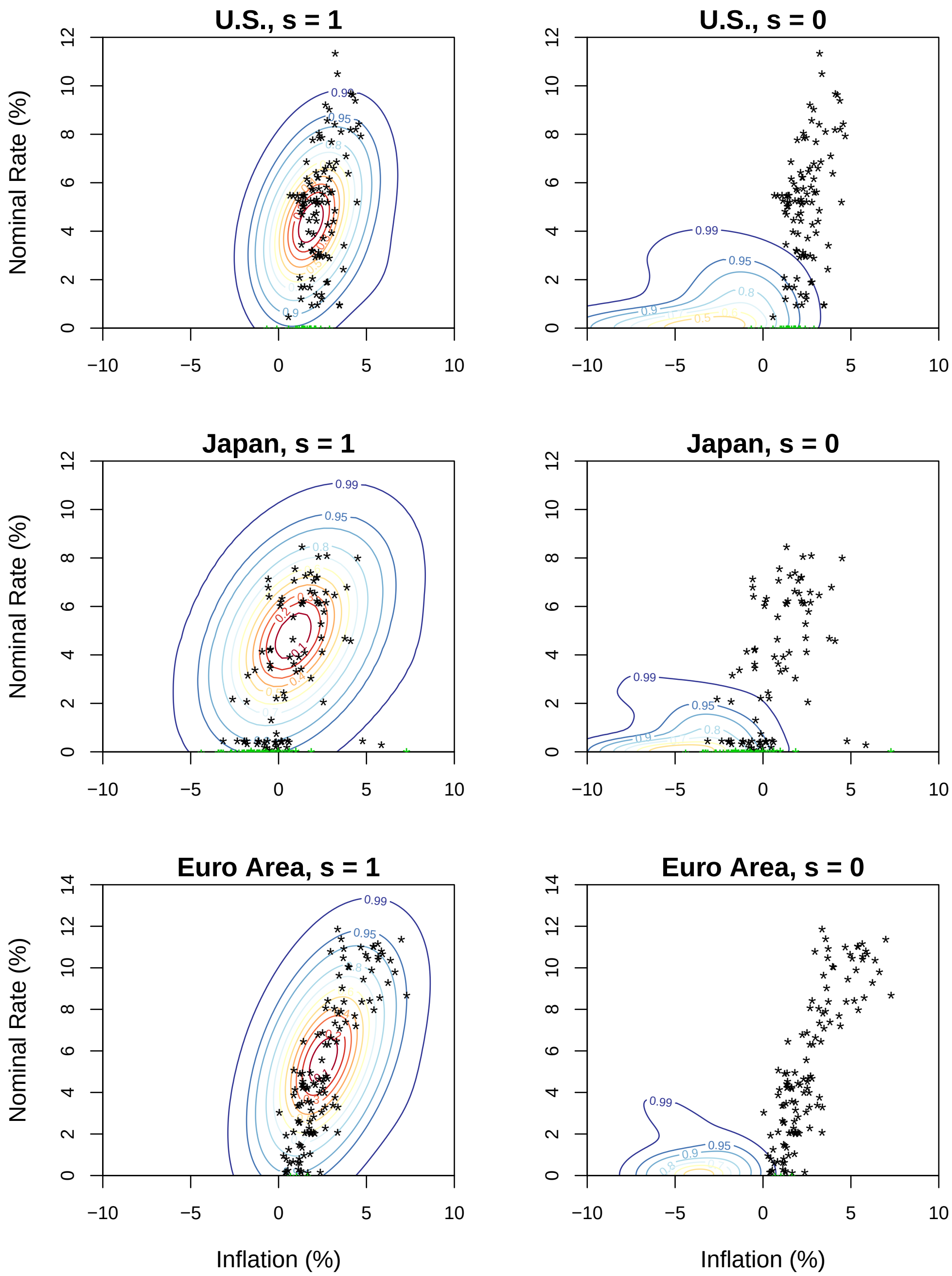
In particular our model has the feature that an economy can reach the ZLB for two different reasons: either adverse yet ultimately transitory shocks within the targeted-inflation regime or a shift to the deflation regime. These two scenarios have very different policy implications and lead to very different predictions about inflation and an exit from the ZLB.

IV Did the U.S., Japan, or the Euro Area Shift to a Deflation Regime?

In ACS we estimate a small-scale DSGE model for the U.S. and Japan under the assumption that the economies are in the targeted-inflation regime, using data that pre-date the ZLB episodes for these two countries. To generate the subsequent results we repeat the estimation for the version of the model presented in Appendix C and also generate estimates for the Euro Area. The parameter values are summarized in Table A-3. To assess whether we have observed a shift to a deflation regime in any of the three economies, we conduct the following experiment: we simulate data from the DSGE models to characterize the joint distribution of interest rates and inflation conditional on the two regimes. We then overlay the observed data to assess whether they appear to be more likely under one of the two regimes. A more formal econometric analysis that utilizes a nonlinear filter is presented in ACS.

Results are presented in Figure 8. The depicted contours in the figure can be interpreted as coverage sets: for instance, the probability that interest rates and inflation fall into the region delimited by the contour labeled 0.95 is 95%. Under the targeted-inflation regime reaching the ZLB is a rare event because it requires an (unlikely) sequence of exogenous

Figure 8: Ergodic Distribution and Data



Notes: In each panel we report the joint probability density function (kernel density estimate) of annualized net interest rate and inflation, represented by the contours. Black stars represent non-ZLB observations: 1984:Q1 - 2008:Q4 (U.S.), 1981:Q1 - 1998:Q4, 2000:Q2-2001:Q1, 2006:Q3-2008:Q4 (Japan), 1984:Q1 - 2014:Q2 (Euro Area). $s = 1$ is the targeted-inflation regime and $s = 0$ the deflation regime. Green stars represent the remaining observations, all which feature the ZLB.

shocks. The probabilities of reaching the ZLB are 0.1%, 0.2%, and 0.2% for the U.S., Japan, and Europe, respectively. A switch to the deflation regime makes it much more likely that the nominal interest rates drop to zero and that we observe negative inflation rates. However, note that especially for the U.S. and Japan, and to some extent for Europe, there is considerable overlap in the regime-conditional distributions: under both regimes it is possible to observe low interest and inflation rates.

The black stars in Figure 8 represent non-ZLB observations for the three economies most of which have been used to estimate the DSGE model parameters. Not surprisingly, they mostly fall within the contours associated with the targeted-inflation regime. More interesting are the green stars, which correspond to near-zero interest rate periods and are excluded from the estimation. For Japan these interest rate and inflation observations appear to be more likely conditional on the deflation regime than under the targeted-inflation regime. For the U.S. the comparison is more ambiguous whereas for the Euro Area a shift to the deflation regime at the current stage looks unlikely to have occurred.

The examination of the contour plots ignores the model's predictions for output and consumption and is no substitute for the formal econometrics analysis conducted in ACS. In ACS we concluded (using a slightly different model) that a sunspot switch did not happen in the U.S., and Japan has been in the deflation regime starting in 1999. While too early to tell (due to limited number of observations) so far Europe seems to stay in the targeted inflation regime as well. In ACS we linked a switch in the sunspot regime to a change in expectations. (Mertens and Ravn (2014) call it a confidence shock.) We concluded that the actions of Bank of Japan following adverse shocks in the late 1990s made the public doubt the central bank's commitment to a positive inflation target and caused a switch in inflation expectations. This lower (and negative) expectations then meant that the economy started fluctuating around the $s = 0$ (deflation) steady state. In contrast, the actions of the Fed following the 2008 financial crisis reassured the public that the positive inflation target is alive and well, and the economy continued to fluctuate around the $s = 1$ (targeted-inflation) steady state.

If we compute inflation expectations from the model, it combines the expectations of the agents in the model regarding a sunspot switch and average inflation in each state. For example, if today we are in the targeted-inflation regime and the agents do not expect to switch to the deflation regime in the foreseeable future, the expected inflation will be close to the targeted inflation. If, however, the public regards a switch as likely (and it will persist for a while) expectations will be lower. This will also be the case where the economy is in the deflation regime today and will exit with some probability in the future, given by its law of motion. Underlying our numerical analysis is the assumption that the regimes are very persistent. A casual look at Figure 1 reveals that for the U.S. and for Europe long-run inflation expectations remain remarkably stable during each country's ZLB episode, while for Japan there is a significant decline following the ZLB episode. This is more evidence that a regime change has occurred in Japan and has not in the U.S. and Europe.

V Low Inflation and Economic Outcomes

Thus far, we have documented that the zero-interest-rate episodes in the U.S., Japan, and the Euro Area are associated with low inflation and, in the case of Japan, with disinflation.

Moreover, looking at the data through the lens of a nonlinear New Keynesian DSGE model, we find some evidence that Japan may have shifted to a what we call deflation regime for an extended period of time. Historically, periods of zero or negative inflation have been associated with low output and high unemployment. The Great Depression of the 1930s and the recent Global Financial Crisis are prominent examples. In the context of DSGE models these crisis are generated by adverse shocks to productivity, aggregate demand, or financial intermediation. Models that are used to study ZLB episodes, feature reduced-form discount factor shocks, e.g., the δ_t in (2), which increase the desire to save, lower current-period consumption demand, and reduce real rates. Of course, these shocks are also a stand-in for other economic mechanisms, e.g., increased risk aversion in times of high uncertainty. Thus, deflation is merely a symptom, but not the cause of poor economic conditions.

Central bankers generally do not like deflation. Many central banks implicitly or explicitly target an inflation rate of about two percent:

> The Federal Open Market Committee (FOMC) judges that inflation at the rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures, or PCE) is most consistent over the longer run with the Federal Reserve's mandate for price stability and maximum employment. Over time, a higher inflation rate would reduce the public's ability to make accurate longer-term economic and financial decisions. On the other hand, a lower inflation rate would be associated with an elevated probability of falling into deflation, which means prices and perhaps wages, on average, are falling–a phenomenon associated with very weak economic conditions. Having at least a small

level of inflation makes it less likely that the economy will experience harmful deflation if economic conditions weaken. The FOMC implements monetary policy to help maintain an inflation rate of 2 percent over the medium term. (Source: *www.federalreserve.gov/faqs/economy_14400.htm.*)

Even though there is no theoretical justification for an inflation target as high as two percent, see Schmitt-Grohé and Uribe (2010), our model embodies the notion that an inflation rate of approximately two percent is important for the public to be able to make accurate longer-term economic and financial decisions.[8] Formally, we assume in the model that it is costly for firms to adjust prices at a rate which differs from the targeted inflation rate. This cost leads to a loss of output in the aggregate, which we call the New Keynesian distortion. While the output loss is not directly observable in the data, in the model it is linked to the slope of the NKPC, which can be estimated. The flatter the NKPC, the larger the output loss. The New Keynesian distortion makes deflation undesirable. For instance, in the simplified version of our DSGE model discussed in Section III.B, welfare in the deflation steady state (in which prices fall at the gross rate of $1/r_*^f$) is substantially lower than in the targeted inflation steady state: one would have to raise consumption in the former by approximately 2.7% to achieve the same level of welfare as in the latter. Of course, if firms would adjust their price-setting technology to the presence of prolonged deflation, the welfare loss would be smaller.

In addition to this New Keynesian channel, downward nominal wage rigidity is often cited as an important reason why deflation is undesirable. While this mechanism is not incorporated into the model that is used in our paper, it is prominently featured in Schmitt-

Grohé and Uribe (2012)'s making of a great contraction with a liquidity trap and a jobless recovery. In the presence of downward nominal wage rigidity, deflation leads to increasing real wages, which depresses employment and output during a recession. While downward rigidity is a well-documented feature of nominal wage changes at the micro level, e.g., Gottschalk (2005), Barattieri, Basu, and Gottschalk (2010), and Daly, Hobijn, and Lucking (2012), making it quantitatively important at the aggregate level is more difficult, because aggregate downward nominal wage rigidity is difficult to measure. The estimates reported in Aruoba, Bocola, and Schorfheide (2013) of the amount of wage rigidity and the asymmetry in the wage adjustment costs are relatively small.

A prominent mechanism that favors low or negative inflation rates is the "Friedman channel," according to which positive nominal interest rates serve as a tax on cash balances, or, more generally, liquid assets that bear negligible interest, and lead agents to economize on transactions involving such assets. Many monetary models without a strong New Keynesian friction prescribe the Friedman rule as the optimal policy. At the steady state, this entails deflation at the rate of time preference. The magnitude of welfare effects depends on how the benefit to consumers and firms of holding cash balances is modeled and how the interest-rate elasticity of money demand is measured. For instance, using a money search model without sticky prices, Aruoba, Waller, and Wright (2011) find that the cost of two percent inflation versus the Friedman rule is about 1.2% of consumption.

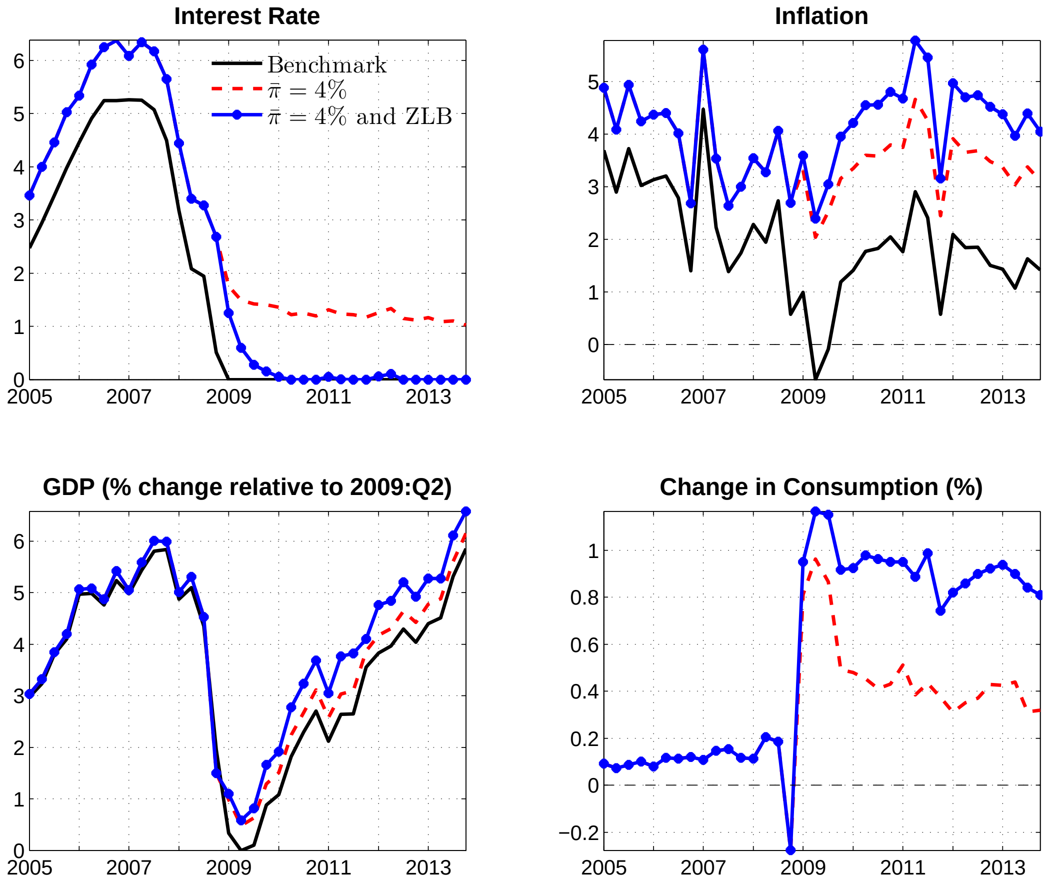
VI Policy Questions

Several prominent economists, e.g., Blanchard, DellAriccia, and Mauro (2010) and Ball (2013), have proposed to raise the inflation target to, for instance, four percent in order to reduce the probability of reaching the ZLB during a period of large adverse shocks. A reappraisal of the targeted inflation rate has remained part of the monetary policy discussions (see Appelbaum (2015)) Using our DSGE model estimated based on U.S. data, under a four percent inflation target, the unconditional probability (simulating the models based on the estimated variance of the structural shocks) of deflation drops from 6.2% to 0.6%. We conduct two counterfactual experiments. In the first experiment, we go back to 1984 and set an inflation target of four percent instead of the estimated (based on average inflation over 1984 to 2007) target of 2.5 percent (Section VI.A). The model is then solved under the assumption that price changes at the rate of four percent are costless, that is, the public accepts this target, views it as credible, and internalizes it in its decisions. In the second experiment, we change the target inflation rate to four percent at the end of 2013 in a way that is understood to be perfectly credible and will remain that way forever (Section VI.B). To generate the counterfactual outcomes, we subject the economy to the same shocks that, according to our benchmark estimation, have occurred during the period from 1984 to 2013.

VI.A What If... the U.S. Had Targeted 4% Inflation?

Figure 9 depicts the path of output, inflation, and interest rates under three scenarios between 2005 and 2013, a period that covers the ZLB episode for the U.S. The first scenario corresponds to the estimated benchmark model with a 2.5% target. By construction, we are

Figure 9: Counterfactual Policy: Long-run Inflation Target of 4%



Notes: Solid black lines correspond to the benchmark policy and reproduce the actual data. Dashed red lines correspond to a counterfactual policy with a target inflation rate of 4% ($\bar{\pi} = 1.01$). Solid-dotted blue lines correspond to a counterfactual target of 4% and a sequence of expansionary monetary policy shocks $\epsilon_{R,t}$ that lower the interest rate to zero. The percentage change in consumption depicted in the bottom right panel is relative to the benchmark policy.

able to recover the actual U.S. data (subject to some small measurement errors) using the estimated shocks. Thus the solid black line in Figure 9 is essentially the U.S. data. Second, we consider a scenario in which the Fed picked 4% as their inflation target in 1984. The path of the key variables under the same structural shocks is given by the red dashed line. A few observations are in order. First, prior to 2009 the main difference between the benchmark scenario and the counterfactual policy are an upward shift of interest and inflation rates by

1.5%. Because we assume that firms adjust their price-setting technology to the new target inflation rate the path of output under the two scenarios is virtually identical up until the end of 2008. Second, after 2008 the ZLB never binds under this counterfactual. Third, inflation never drops below zero and promptly returns near the target of the Fed. Fourth, the recovery in GDP is somewhat faster.

A non-binding ZLB between 2009 and 2014 would have given the Fed the ability to conduct conventional expansionary monetary policy by lowering interest rates to zero. We consider such a policy in our third scenario. Using a sequence of unanticipated monetary policy shocks $\epsilon_{R,t}$, we reduce the nominal interest rate to zero under the 4% inflation target. The path of variables under this scenario is shown by blue circles in Figure 9. Here the return of inflation to average levels is even quicker and recovery of GDP takes about a year less than under the historical policy.

The last panel in Figure 9 shows that under both of the scenarios consumption is substantially higher relative to the benchmark after 2009. This may imply that welfare is higher as well. However, one should keep in mind that welfare takes in to account both consumption and leisure, and taken as a whole, the comparison of welfare in the 2008-2014 period is ambiguous. Even if we conclude, at least qualitatively, that there are welfare gains to a higher inflation target during an episode where the ZLB would have bound otherwise, the overall benefits of this policy are far from clear.

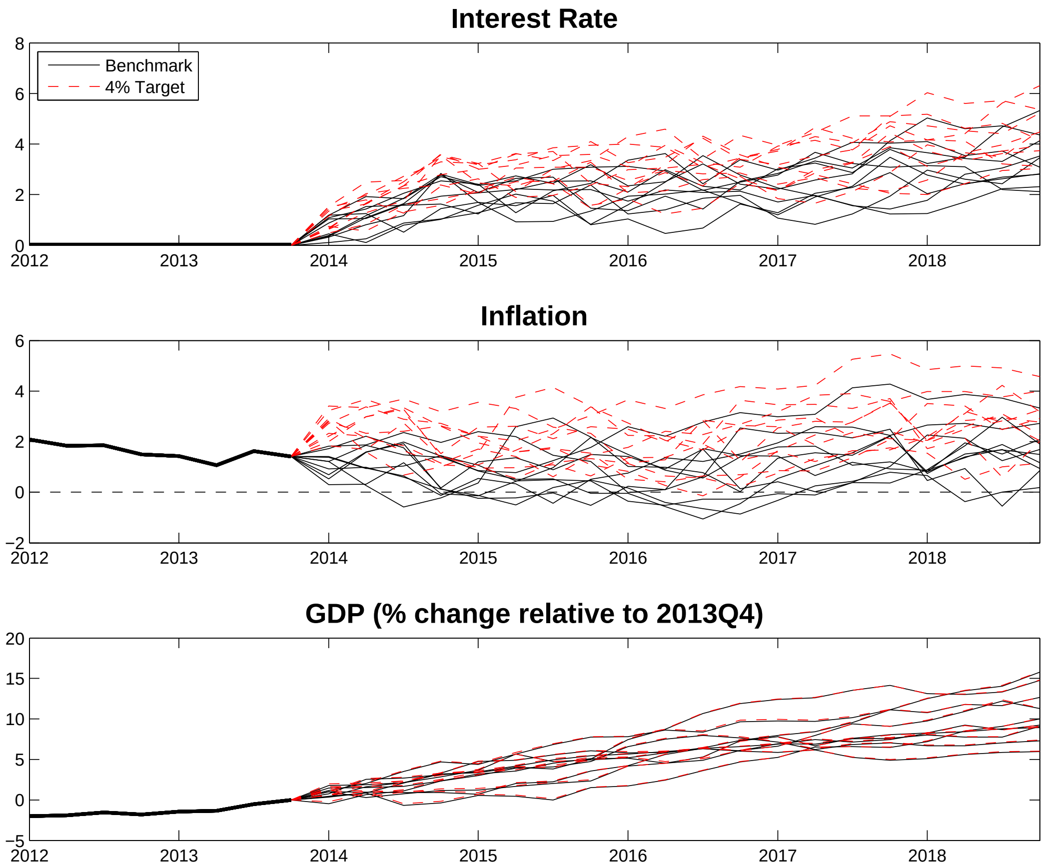
Our analysis assumed that the public adjusts to this level of average inflation and thus the New Keynesian channel is mute. If this was not the case, that is, if the required average price adjustments due to a higher target inflation rate created a cost, then there would be an output and welfare loss associated with the New Keynesian channel. This is an important concern. For instance, the head central bankers of Germany and Switzerland, see Weber and Hildebrand (2010), argue that changing the inflation target would destroy the credibility they built regarding their commitment to price stability. Finally, as we explained above, the Friedman channel may contribute to additional welfare costs during "normal" times. For example, using the calculations in Aruoba and Schorfheide (2011), which account for both the New Keynesian and the Friedman channel, the welfare loss of changing the long-run inflation to 4% from 2.5% is about 0.6% of consumption.

Finally, the overall gains depend crucially on the probability of ever getting to the ZLB with a lower target. As we mention, this probability is very small in our estimated model – less than 0.1% for the U.S. Some authors argue that the fact that Japan has experienced ZLB for as long as it did is evidence that ZLB is not that rare. However, our analysis in ACS shows that Japan has experienced an extended period of deflation for a different reason, namely, a switch into the deflation regime. At least within the logic of our model, this switch is unrelated to the target inflation rate and could also occur for a target of four percent.

VI.B What If... the U.S. Switches to a 4% Target Now?

We now consider a hypothetical switch to a 4% target rate in 2014:Q1, conditioning on the state of the U.S. economy at the end of 2013:Q4. Results are depicted in Figure 10. We show 10 random trajectories under the two scenarios that share the same underlying structural innovations. First, notice that even under the benchmark target, the model predicts a lift-off from the ZLB. This prediction is common to many DSGE models, indicating that the

Figure 10: Counterfactual Policy: Inflation Target of 4% in 2014



Notes: The black lines prior to 2014:Q4 represent actual U.S. data. The subsequent (black) hairs correspond to simulated trajectories under the prevailing policy. The red dashed lines correspond to simulated trajectories (based on the same sequence of stochastic disturbances) under the counterfactual 4% target.

current monetary policy is, by historical standards, unusually expansionary.[9] According to the estimated model, adverse shocks pushed the economy to the ZLB but, based on historical experience, these shocks tend to be mean reverting, which is consistent with the observed (albeit slower than expected) recovery.

Second, the interest rate, output, and inflation forecasts reflect substantial uncertainty. Under the benchmark scenario there remains a risk of deflation as late as 2017, which is broadly in line with the forecasts presented from the local-level model presented in Figure 2.

Third, the lift-off from the ZLB is faster under the four percent target inflation rate and the deflation risk is reduced. Finally, while the change in the target inflation rate affects interest rate and inflation dynamics, the path of GDP is largely unaffected. Thus, this analysis suggests that if the central bank raises the inflation target now, even if it is able to communicate and convince the public about the credibility of this new policy, there does not seem to be much real effects of this policy change to make it desirable.

VI.C Other Policies

The literature has discussed many other policies in the context of the ZLB, in particular unconventional monetary policies such as large-scale asset purchases, often called quantitative easing, the effects of forward guidance signaling an extended period of low interest rates, and a switch from inflation to price level targeting as a way of creating a commitment to expansionary monetary policy after a period of low, possibly negative, inflation rates. Because our model abstracts from frictions that interact with these policies, e.g., limited asset market participation of some households and firms, or informational frictions that affect the credibility of central bank announcements, we do not analyze the effect of these policies here.

Throughout this paper we have stressed multiplicity of equilibria in workhorse New Keynesian DSGE models. We now provide a brief discussion of policies that interact with these multiplicities. First, the deflation steady state could be eliminated with a policy that drastically raises the interest rate if inflation falls below some threshold and thereby eliminates the intersection of the Fisher equation and the monetary policy rule at the ZLB. However, introducing such a policy at a point in time when an economy is at the ZLB and potentially experiences deflation is problematic. As we have documented in Section IV, in real time it is difficult to determine whether an economy is at the ZLB due to a sequence of adverse shocks in the targeted-inflation regime, or whether it is at the ZLB because of a shift to the deflation regime. Switching to a new policy rule that implies a drastic increase in interest rates is very costly in the first case because it will deepen the recession.

The deflation steady state would also disappear under a passive monetary policy with $\psi_1 < 1$. The downside of such a monetary policy is that, in combination with a passive fiscal policy in the terminology of Leeper (1991), it leaves the fluctuations of output, inflation and interest rates around the targeted-inflation steady state indeterminate. Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004) document that passive monetary policy may have been one of the culprits behind high inflation rates and high macroeconomic volatility in the 1970s. A compromise could be a fiscal policy that responds to inflation or the nominal value of government debt. We stressed in Section III.B.i that in both the deflation and the targeted-inflation steady states one can sustain a constant real value of government debt because the real interest rate is identical in the two steady states. However, in nominal terms, debt keeps on falling in the deflation steady state, whereas it rises in the targeted inflation steady state. A fiscal policy that, for instance, is committed to lowering real taxes in response to the level of nominal debt would make the deflation steady state fiscally unsustainable. An implication of the work by Davig and Leeper (2007) is that active fiscal policy in times when inflation and interest rates are low can also eliminate local indeterminacies at the ZLB.

Our DSGE model featured an exogenous sunspot process that determined the inflation regime. We used this sunspot shock as a substitute for a theory of equilibrium selection.

It served as a coordination device for agents in the model. In reality it is conceivable that a central bank has considerable influence on this expectation coordination through its communication. In fact, in ACS we argue that the aggressive unconventional monetary policies in the U.S., in contrast to the more measured responses of the Bank of Japan, may have prevented a switch to the deflation regime in the U.S.

VII Conclusion

In this paper we tried to shed some light on how inflation dynamics may change when an economy hits the zero lower bound of interest rates. We considered the experiences of Japan, the U.S., and the Euro Area through the lens of a univariate time series model on the one hand and a New Keynesian DSGE model on the other hand. It turns out that the predictions of the workhorse DSGE model are ambiguous, because multiple equilibria can arise. The multiplicity is a blessing and a curse. It allows us to rationalize disparate cross-country experiences but it also generates a lot of uncertainty about the effect of economic policies. Policies that eliminate multiplicity of potential outcomes or coordinate households and firms expectations and actions on the desired outcome, should have a high priority.

Appendix

A Data

A.1 United States

Real per capita GDP: We obtained real GDP (GDPC96) and converted into per capita terms using the Civilian Noninstitutional Population (CNP16OV). The population series is smoothed applying an eight-quarter backward-looking moving average filter. Source: FRB St. Louis FRED database.

Real per capita consumption: We obtained real personal consumption expenditures (PCECC96) and converted into per capita terms using the Civilian Noninstitutional Population (CNP16OV). Source: FRED.

GDP Deflator Inflation: computed as log difference of GDP deflator (GDPDEF), multiplied by 400 to convert it into annualized percentages. Source: FRED.

CPI Inflation: computed as log difference of CPI (CPIAUCSL), multiplied by 400 to convert it into annualized percentages. Source: FRED.

Interest Rate / Monetary Policy Rate: effective federal funds rate (FEDFUNDS) averaged over each quarter. Source: FRED.

Inflation Expectations: 1-year-ahead and 5-year-ahead inflation expectations from Aruoba (2014) averaged over each quarter.

A.2 Japan

Real per capita GDP: We collected real GDP (RGDP) from the Cabinet Office's National Accounts. We used the statistical release of benchmark year 2005 that covers the period 1994.Q1 - 2013.Q4. To extend the sample we collected RGDP figures from the benchmark year 2000 and constructed a series spanning the period 1981.Q1-2013.Q1 using the quarterly growth rate of the RGDP benchmark year 2000. Our measure of per-capita output is RGDP divided by the total population of 15 years and over. We smoothed the quarterly growth of the population series using an eight quarter backward-looking moving average filter. We obtained population data from the Statistics Bureau of the Ministry of Foreign Affairs Historical data Table b-1.

Real per capita consumption: We collected real Private Consumption data from the Cabinet Office's National Accounts and follow the same procedure as for real GDP to convert it into per capita terms.

GDP deflator inflation: For the price level we use the implicit GDP deflator index from the Cabinet Office. We also extend the benchmark year 2005 release using the growth rate of the index from the benchmark year 2000 figures.

Interest Rate / Monetary Policy Rate: For the nominal interest rate we use the Bank of Japan's uncollateralized call rate (STSTRACLUCON) from 1986:M7-2013:M12. To complete the series from 1981.M1 - 1985.M6 we use the monthly average of the collateralized overnight call rate (STSTRACLCOON). Finally the monthly figures are transformed using quarterly averages over the sample period.

Inflation Expectations: 10-year-ahead inflation expectations are obtained from iMFdirect March 4, 2014 post "Euro Area - Deflation versus Lowflation" by Moghadam, Teja, and Berkmen. As 1-year-ahead inflation expectations we use December Blue Chip forecasts for the following year. Both of these measures are observed at an annual frequency.

A.3 Euro Area

Real GDP: YER. Source: *Area Wide Model* database, see ECB Working Paper No. 42.

Real Consumption: PCR. Source: *Area Wide Model* database.

GDP Deflator Inflation: computed as log differences of YED, scaled by 400. Source: *Area Wide Model* database.

CPI Inflation: computed as log differences of HICP, scaled by 400. Source: *Area Wide Model* database.

Interest Rate: short-term interest rate (STN). Source: *Area Wide Model* database.

Monetary Policy Rate: interest rate on the main refinancing operations (MRO). Source: ECB.

Inflation Expectations: 1-year-ahead and 5-year-ahead inflation forecasts. Source: ECB Survey of Professional Forecasters.

B Estimation of the Local-Level Model

We estimate the local-level model (1) based on GDP deflator inflation data from the U.S., Japan, and the Euro Area over the period from 1984:Q1 to 2014:Q4 using Bayesian techniques designed for state-space models with stochastic volatility. The prior distribution is

Table A-1: Prior Distribution for Local Level Model

	U.S.	Japan	Euro Area
φ	U[0,1]	U[0,1]	U[0,1]
σ	IG(3,2.5)	IG(3,5)	IG(3,4.5)
ν_η	N(0.9, 0.5)	N(0.9, 0.5)	N(0.9, 0.5)
ν_ϵ	N(0.9, 0.5)	N(0.9, 0.5)	N(0.9, 0.5)
σ_{ν_η}	IG(3, 0.1)	IG(3, 0.1)	IG(3, 0.1)
σ_{ν_ϵ}	IG(3, 0.01)	IG(3, 0.01)	IG(3, 0.01)

Table A-2: Posterior Medians and 90% Credible Intervals for Local Level Model

	U.S.		Japan		Euro Area	
φ	0.59	[0.34, 0.88]	0.17	[0.09, 0.29]	0.53	[0.37, 0.77]
σ	0.53	[0.42, 0.66]	1.55	[1.55, 1.55]	0.65	[0.56, 0.77]
ν_η	0.43	[-0.42, 0.91]	0.44	[-0.34, 0.94]	0.48	[-0.40, 0.92]
ν_ϵ	0.74	[-0.23, 0.98]	0.51	[-0.98, 0.93]	0.56	[-0.32, 0.97]
σ_η	0.24	[0.14, 0.53]	0.24	[0.13, 0.54]	0.25	[0.14, 0.52]
σ_ϵ	0.12	[0.05, 0.39]	0.17	[0.056, 0.45]	0.09	[0.04, 0.27]

summarized in Table A-1. Note that Inverse Gamma distribution $IG(a, b)$ is parameterized as $p_{IG}(\sigma \mid a, b) \propto \sigma^{-a-1} \exp(b/\sigma)$. We use different priors for σ across countries. The median of the prior is chosen to match a pre-sample sample standard deviation of inflation.

C DSGE Model

C.1 Households, Firms, Government Policies, and Shocks

The empirical analysis in this paper is based on the following DSGE model:

Households: solve the following problem:

$$\max_{C_t,H_t,B_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \delta_t \left(\frac{(C_t/A_t)^{1-\tau} - 1}{1-\tau} - \frac{H_t^{1+1/\eta}}{1+1/\eta} \right) \right], \tag{A.1}$$

subject to:

$$P_t C_t + B_t + T_t = W_t H_t + R_{t-1} B_{t-1} + P_t D_t + P_t SC_t.$$

Here β is the discount factor, δ_t is a discount factor shock, C_t is consumption, which enters the utility functions relative to the level of technology A_t, H_t is hours worked. The budget constraint is written in nominal terms: P_t is the price of the final good, B_t are government bonds, T_t are taxes, W_t are nominal wages, R_t is the nominal interest rate, D_t are dividend payments from the firms, and SC_t net proceeds from trading state-contingent claims.

Firms: Perfectly competitive, final goods producing firms combine a continuum of intermediate goods indexed by $j \in [0, 1]$ using the technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \tag{A.2}$$

Here $1/\nu > 1$ represents the elasticity of demand for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given. Profit maximization implies that the demand for intermediate goods is:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t. \tag{A.3}$$

Free entry implies that the relationship between intermediate goods prices and the price of the final good is:

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \tag{A.4}$$

Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \tag{A.5}$$

where A_t is an exogenous productivity process that is common to all firms. Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of the firm's output, are defined by the function

$$\Phi_p(x) = \phi(x - \bar{\pi})^2. \tag{A.6}$$

Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits:

$$\begin{aligned} \max_{\{H_{t+s}(j),P_{t+s}(j)\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \delta_{t+s} Q_{t+s|t} \Bigg(& \frac{P_{t+s}(j)}{P_{t+s}} A_{t+s} H_{t+s}(j) \\ & -\Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) A_{t+s} H_{t+s}(j) - \frac{W_{t+s} H_{t+s}(j)}{P_{t+s}} \Bigg), \end{aligned} \tag{A.7}$$

subject to

$$A_t H_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t.$$

Monetary and Fiscal Policies: Monetary policy is described by the interest rate feedback rule defined in (11) and (12). The fiscal authority consumes a fraction ζ_t of aggregate output Y_t, where $\zeta_t \in [0, 1]$ follows an exogenous process. The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t, \tag{A.8}$$

where $G_t = \zeta_t Y_t$.

Exogenous Shock Processes: The model economy is perturbed by four exogenous processes:

$$\begin{aligned} \ln A_t &= \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \text{where} \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t} \\ \ln g_t &= (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} \\ \ln \delta_t &= \rho_\delta \ln \delta_{t-1} + \epsilon_{\delta,t}, \end{aligned} \tag{A.9}$$

where $g_t = 1/(1-\zeta_t)$, and the monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated.

C.2 Equilibrium Conditions

We use the following stationarity inducing transformations: $y_t = Y_t/A_t$, and $c_t = C_t/A_t$. We also define the gross inflation rate $\pi_t = P_t/P_{t-1}$. The equilibrium conditions are given by

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \left(\frac{\delta_{t+1}}{\delta_t} \right) \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{A.10}$$

$$-1 + \frac{1}{\nu}\left(1 - c_t^\tau y_t^{1/\eta}\right) + \phi(\pi_t - \bar{\pi})\left[\left(1 - \frac{1}{2\nu}\right)\pi_t + \frac{\bar{\pi}}{2\nu}\right] \tag{A.11}$$

$$= \phi \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \left(\frac{\delta_{t+1}}{\delta_t} \right) \pi_{t+1} (\pi_{t+1} - \bar{\pi}) \frac{y_{t+1}}{y_t} \right]$$

$$R_t = \left[r\pi^* \left(\frac{\pi_t}{\pi^*} \right)^{\psi_1} \left(\frac{Y_t}{\bar{Y}_t} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} \exp(\epsilon_{R,t}) \tag{A.12}$$

$$c_t = \left(\frac{1}{g_t} - \frac{\phi}{2}(\pi_t - \bar{\pi})^2 \right) y_t \tag{A.13}$$

C.3 A Simplified Version of the Model

In the main text we refer to a simplified version of the DSGE model which is obtained by setting $\tau = 1$, $\eta = \infty$, $\psi_2 = 0$, $\rho_R = 0$. In the targeted-inflation steady state we have

$$\pi_* = \bar{\pi}, \quad r^f_* = \gamma/\beta, \quad R_* = r^f_* \bar{\pi}, \quad c_* = 1 - \nu, \quad y_* = g_* c_*. \tag{A.14}$$

The deflation steady state is given by

$$\begin{aligned} \pi_* &= 1/r^f_*, \quad r^f_* = \gamma/\beta, \quad R_* = 1 \qquad \text{(A.15)} \\ c_* &= 1 - \upsilon - \frac{\phi}{2}(1-2\lambda)\left(\pi_* - \frac{1-\lambda}{1-2\lambda}\bar{\pi}\right)^2 + \frac{\phi}{2}\frac{\lambda^2}{1-2\lambda}\bar{\pi}^2 \\ y_* &= \frac{c_*}{\frac{1}{g_*} - \frac{\phi}{2}(\pi_* - \bar{\pi})^2}, \end{aligned}$$

where $\lambda = \nu(1-\beta)$.

We also refer to the log-linearized equilibrium conditions (around the targeted-inflation steady state), which are given by

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - (\widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1} + \widehat{z}_{t+1} - \widehat{\delta}_{t+1} + \widehat{\delta}_t]) \tag{A.16}$$

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{mc}_t, \quad \text{where} \quad \widehat{mc}_t = \widehat{c}_t, \ \kappa = c_*/(\nu\phi\bar{\pi}^2) \tag{A.17}$$

$$\widehat{R}_t = \max\left\{-\ln R_*, \ \psi_1 \widehat{\pi}_t + \epsilon_{R,t}\right\} \tag{A.18}$$

$$\widehat{y}_t = \widehat{c}_t + \widehat{g}_t. \tag{A.19}$$

Here $\widehat{x}_t = \ln(x_t/x_*)$.

C.4 Derivations for Section III.B

This is a discrete-time version of the calculations in Cochrane (2015). To simplify the notation we omit hats from the variables. Let $Rr_t = R_t - r_t$. Then the perfect foresight system can be written as

$$c_t = \mathbb{E}[c_{t+1}] - (Rr_t - E[\pi_{t+1}])$$
$$\pi_t = \beta E[\pi_{t+1}] + \kappa c_t.$$

To iterate the system forward, we express time $t+1$ variables as functions of time t variables. In matrix form, the system becomes: This leads to

$$\begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t.$$

Solving for (c_{t+1}, π_{t+1}) we obtain

$$\begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\kappa/\beta & -1/\beta \\ -\kappa/\beta & 1/\beta \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t = \Gamma_* \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t. \quad \text{(A.20)}$$

We proceed by calculating the eigenvalues of the autoregressive matrix Γ_*. Define $\rho = 1/\beta$. This amounts to solving the quadratic equation

$$\begin{aligned} 0 &= (1+\kappa\rho-\lambda)(\rho-\lambda) - \kappa\rho^2 \\ &= \lambda^2 - \lambda(1+\kappa\rho+\rho) + (1+\kappa\rho)\rho - \kappa\rho^2 \\ &= \lambda^2 - \lambda(1+\rho(1+\kappa)) + \rho. \end{aligned}$$

The solutions are

$$\begin{aligned} \lambda_1 &= \frac{1+\rho(1+\kappa)}{2} + \sqrt{\frac{(1+\rho(1+\kappa))^2}{4} - \rho} \\ \lambda_2 &= \frac{1+\rho(1+\kappa)}{2} - \sqrt{\frac{(1+\rho(1+\kappa))^2}{4} - \rho}. \end{aligned}$$

Note that $\rho > 1$ and $\kappa > 0$, which implies that

$$\frac{1+\rho(1+\kappa)}{2} > 1.$$

Moreover

$$\begin{aligned}(1+\rho(1+\kappa))^2 - 4\rho &= 1 + 2\rho + 2\rho\kappa + \rho^2(1+\kappa)^2 - 4\rho \\ &= (\rho-1)^2 + \kappa\rho(2+\rho+\kappa\rho) \\ &> 0.\end{aligned}$$

We conclude that λ_1 is an unstable eigenvalue.

Now note that

$$\sqrt{\frac{(1+\rho(1+\kappa))^2}{4} - \rho} < \sqrt{\frac{(1+\rho(1+\kappa))^2}{4}} = \frac{1+\rho(1+\kappa)}{2},$$

which implies that $\lambda_2 > 0$. In order to show that $\lambda_1 < 1$, we need to show that

$$\frac{\rho-1}{2} + \frac{\rho\kappa}{2} \leq \frac{1}{2}\sqrt{(\rho-1)^2 + \rho^2\kappa^2 + 2\rho\kappa(1+\rho)}.$$

Multiplying by 2 and squaring both sides of the equation yields

$$(\rho-1)^2 + \rho^2\kappa^2 + 2\rho\kappa(\rho-1) < (\rho-1)^2 + \rho^2\kappa^2 + 2\rho\kappa(\rho+1).$$

Thus, we verified that $0 \leq \lambda_2 < 1$.

Now consider the eigenvalue decomposition of the matrix Γ_*, which we write as as $\Gamma_* J\Lambda J^{-1}$. We can now define $w_{t+1} = J^{-1}[c_{t+1}, \pi_{t+1}]'$. Let $(J^{-1})_{1.}$ be the first row of J^{-1}, which corresponds to the eigenvector associated with the unstable root λ_1. To ensure that the system is stable for $t > T$ conditional on $Rr_t = 0$, it has to be the case that

$$J^{-1}_{1.}\begin{bmatrix} c_{T+1} \\ \pi_{T+1} \end{bmatrix} = 0, \tag{A.21}$$

which determines c_{T+1} as a function of π_{T+1}. Figure 7 is generated as follows (i) choose π_{T+1}; (ii) solve (A.21) for c_{T+1}; (iii) iterate (A.20) forward for $t > T+1$; (iv) iterate backward using

$$\begin{bmatrix} c_t \\ \pi_t \end{bmatrix} = \Gamma_*^{-1} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} - \Gamma_*^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t. \tag{A.22}$$

for $t \leq T$.

C.5 Parameterization of DSGE Models

The parameters for the DSGE model-based analysis are obtained as follows: (1) We calibrate γ, β, $\bar{\pi}$, g_*, η, ψ_1, ψ_2, ν, p_{00}, and p_{11}. The steady-state related parameters are calibrated based on long-run averages. (2) We use Bayesian techniques to estimate the remaining parameters. The estimation periods are: 1984:Q1 - 2007:Q4 (U.S.); 1981:Q1 - 1994:Q4 (Japan); 1984:Q1 - 2007:Q4 (Euro Area). The parameter values are summarized in Table A-3.

Table A-3: DSGE Model Parameters

Parameters	Description	U.S.	Japan	Euro Area
$100 \ln \gamma$	Quarterly growth rate of technology	0.496	0.565	0.574
$400(1/\beta - 1)$	Annualized discount rate	0.861	1.878	0.930
$400 \ln \bar{\pi}$	Annualized inflation rate	2.465	1.278	3.102
$(C/Y)_*$	SS consumption/output ratio	0.647	0.579	0.567
τ	Inverse IES	1.993	1.641	2.119
η	Frisch elasticity	0.720	0.850	0.791
ν	EOS intermediate inputs	0.100	0.100	0.100
κ	Slope (linearized) Phillips curve	0.101	0.425	0.525
ψ_1	Taylor rule: weight on inflation	1.500	1.500	1.500
ψ_2	Taylor rule: weight on output growth	0.100	0.100	0.100
α	Smoothing coeff. for trend output	0.900	0.850	0.630
ρ_R	Interest rate smoothing	0.799	0.745	0.737
ρ_d	Persistence: discount shock	0.954	0.906	0.957
ρ_g	Persistence: demand shock	0.955	0.928	0.981
ρ_z	Persistence: technology shock	0.188	0.086	0.098
$100\sigma_R$	Std dev: monetary policy shock	0.160	0.190	0.160
$100\sigma_d$	Std dev: discount shock	1.880	1.180	1.620
$100\sigma_g$	Std dev: demand shock	0.530	0.770	0.400
$100\sigma_z$	Std dev: technology shock	0.500	1.090	0.450
p_{00}	Prob of staying in deflation regime	0.975	0.975	0.975
p_{11}	Prob of staying in targeted-inflation regime	0.990	0.990	0.990

Notes: Note that $g_* = 1/(C/Y)_*$.

Endnotes

[1]Models with multiple equilibria are common in many areas of economics. For instance, an important example in the industrial organization literature is an entry game model with two potential entrants. For markets that can support a profitable monopoly but not a profitable duopoly the model tends to be silent about which firm enters the market.

[2]The first of these spikes is in 2008:Q4 and corresponds to a massive decline in imports during the global financial crisis that skews GDP deflator up. The second one is in 2014:Q2 and it corresponds to a one-time increase in value-added tax. Neither of these spikes show up in CPI inflation.

[3]This statement is subject to the following caveat. The price setting equation underlying the DSGE model is nonlinear. If the ZLB episodes are associated with low inflation, then the approximate price dynamics change. For instance, if we set $\tau = 1$ and the remaining parameters according to Table A-3, then NKPC takes the form $\widehat{\pi}_t = 0.998\mathbb{E}_t[\widehat{\pi}_{t+1}] + 0.051\widehat{mc}_t$ if inflation is close to 2.5% and $\widehat{\pi}_t = 0.919\mathbb{E}_t[\widehat{\pi}_{t+1}] + 0.048\widehat{mc}_t$ if inflation is close to -2.8%.

[4]In the context of the model described in Appendix C the real rate process is given by $\widehat{r}_t = \rho_z z_t - (\rho_d - 1)\delta_t$, where ρ_z and ρ_d are the autocorrelations of the technology growth and the discount factor process.

[5]There is some disagreement how to handle dynamics under which inflation is explosive but real consumption and output are not. Cochrane (2011) argues that such paths should not be ruled out, while other researchers tend to rule them out.

[6]A Textbook treatment can be found, for instance, in Woodford (2003).

[7]A similar analysis can be conducted for the case of $1/\kappa > 0$. However, in this case the transition is instantaneous because the system given by (18) and (19) has two unstable roots and one root that is equal to zero. Once one sets the linear combinations of interest rates, inflation, and consumption associated with the unstable eigenvalues to zero, the linear combination given by the third eigenvector adjusts instantaneously because the third eigenvalue is zero.

[8]For U.S. data we set the target inflation rate in our model to 2.5% instead of 2% because the former number corresponds to the average GDP deflator inflation rate over our estimation sample.

[9]See also the discussion in Del Negro and Schorfheide (2013) of this phenomenon and how external interest rate forecasts in combination with anticipated monetary policy shocks are needed to forecast a prolonged period of zero interest rates with standard DSGE models.

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